Energy Dissipation of Axionic Boson Stars in Magnetized Conducting Media

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Abstract
Axions are possible candidates of dark matter in the present Universe. They have been argued to form axionic boson stars. Since they are shown to possess oscillating electric fields in a magnetic field, they lose their energies in magnetized conducting media. We show that colliding with a white dwarf, the axionic boson stars dissipate their energies with rate being roughly $\sim 10^{50}$ erg/s. According to recent evaluation of the population of the white dwarfs as candidates of MACHOs, we estimate that the event rate of the collisions is roughly 4 per year in a solid angle $5^\circ \times 5^\circ$.

The axion is the Goldstone boson associated with Peccei-Quinn symmetry [1], which was introduced to solve naturally the strong CP problem. In the early Universe some of the axions condense and form topological objects [2, 3], i.e. strings and domain walls, although they decay below the temperature of QCD phase transition. After their decay, however, they have been shown to leave a magnetic field [4] as well as cold axion gas as relics in the present Universe; the field is a candidate of a primordial magnetic field leading to galactic magnetic fields observed in the present Universe.

In addition to these topological objects, the existence of axionic boson stars has been argued [5, 6]. It have been shown numerically [6] that in the early Universe, axion clumps are formed around the period of 1 GeV owing to both the nonlinearity of an axion potential and the inhomogeneity of coherent axion oscillations on the scale beyond the horizon. These clumps are called axitons since they are similar to solitons in a sense that its energy is localized. Then, the axitons contract gravitationally to axion miniclusters [7] after separating out from the cosmological expansion. They are compact bosonic objects contrary to rather uniformly distributed axion gas generated by the axion strings or the coherent axion oscillations. Furthermore, depending on their energy densities, some of these miniclusters may contract gravitationally to coherent boson stars [8, 6, 9]. Eventually we expect that in the present Universe, there exist the axion miniclusters and the axion boson stars as well as the incoherent axion gas as dark matter candidates. It has been estimated [10] that a fairly amount of the fraction of the axion dark matter comprises these axion clumps. An observational implication of the axion miniclusters has been discussed [10].

In this letter we wish to point out an intriguing observable effect associated with these coherent axionic boson stars; we call them axion stars. Namely, they release their energies in magnetized conducting media so much that resultant radiations are observable. As we will show below, the coherent axion stars generate electric fields in external magnetic fields. These electric fields induce electric currents in the conducting media and loose their energies owing to existence of resistances. Although the electric fields themselves are small as expected, the amount of the energy dissipation is
very large because the dissipation arises over the large volume of the axion stars. Consequently strong detectable radiations are expected from the media heated. Because the strength of the electric fields is proportional to the strength of the magnetic field, the phenomena are distinctive especially in strongly magnetized media such as neutron stars, white dwarfs e.t.c.. We show that the amount of the energy released in white dwarfs with mass $\sim 0.5 \, M_\odot$ and with magnetic field larger than $10^8$ Gauss is approximately $10^{36} \, \text{erg/s} \left( M/10^{-14} M_\odot \right)^4 \left( m/10^{-3} \text{eV} \right)^6$ where $M(M_\odot)$ is the mass of the axion star (the sun) and $m$ is the mass of the axion.

Let us we explain how the coherent axion field generates an electric field in an external magnetic field. The point is that the axion couples with the electric magnetic fields in the following way,

$$L_{\alpha \gamma} = c \alpha \, a \vec{E} \cdot \vec{B} / f \pi$$

with $f$ being the axion decay constant and $\alpha = 1/137$, where $a$ is axion field and, $\vec{E}$ and $\vec{B}$ are electric and magnetic fields respectively. The value of $c$ depends on the axion models [11, 12]; typically it is the order of one. The value of $f$ is constrained from cosmological and astrophysical considerations [3], $10^9 \, \text{GeV} < f < 10^{12} \, \text{GeV}$. It is easy to see from this interaction that the coherent axion field may have electric charge density $- c \alpha \, a \vec{E} \cdot \vec{B} / f \pi$ in the magnetic field $\vec{B}$ [13]. We assume that the field $\vec{B} = (0,0, B)$ is spatially uniform and that the geometry of the axion field $a$ representing the boson stars is spherical. Then we understand easily that this axion object has a charge distribution such that it has negative charges on a hemisphere ($z > 0$) and positive charges on the other side of the sphere ($z < 0$). Net charge is zero. Therefore the star possesses the electric field, $E$, parallel to the magnetic field associated with the charge distribution; $E = c \alpha \, a B / f \pi$. This field induces an electric current in conducting media.

Denoting the conductivity of the media by $\sigma$ and assuming the Ohm law, we can see easily that the axion star with it's radius $R$ releases an energy $W$ per unit time,

$$W = \sigma \, c^2 \, e^2 a^2 B^2 a_0^2 R^4 / \pi = \alpha^2 \, c^2 B^3 a_0^2 R / 4 \pi^2 \nu_m$$,

where we have assumed that the distribution of the axion field representing a boson star is given such that $a = f a_0 \exp \left( - r / R \right)$; $r$ denotes a radial coordinate. We have taken account of the fact that the field a oscillates [14, 15] with a frequency given approximately by the mass of the axion $m; a \propto \sin mt$. Hence we have taken an average in time over the period, $m^{-1}$. $\nu_m = 1 / 4 \pi \sigma$ is magnetic diffusivity.

We comment that the formula may apply to the conducting media where the Ohmic law is hold even for oscillating electric fields with their frequencies $m = 10^9 \sim 10^{11} \, \text{Hz}$. The law is hold in the media where electrons interact sufficiently many times in a period of $m^{-1}$ with each others or charged particles in environment, and diffuse their energies acquired from the electric field. Actually the law is hold in the convection zone of the sun, white dwarfs, neutron stars e.t.c..

In order to see the existence of axionic boson stars, we have obtained numerically solutions of the axionic boson stars [14, 15] in a limit of a weak gravitational field by solving a free field equation of
the real scalar field $\phi$ along with Einstein equations. It means that our solutions represent the axion stars with small masses, e.g. $10^{-12} M_\odot$ whose gravitational fields are sufficiently weak and amplitudes $a_0$ are much small. We have confirmed that the spherical distribution of the field assumed above is hold practically. In the limit of the small mass of the axion star we have found a relation [14] between the mass, $M$ and the radius, $R$ of the axion star,

$$M = 6.4 \frac{m_p^2}{m^2 R} ,$$

with Planck mass $m_p$. Numerically we can see that for example, $R = 1.6 \times 10^5 m_5^{-2}$ cm for $M = 10^{-9} M_\odot$, $R = 1.6 \times 10^8 m_5^{-2}$ cm for $M = 10^{-12} M_\odot$, e.t.c. with $m^5 \equiv m/10^5$ eV.

We have also found a relation [14] between the radius and the amplitude, $a_0$,

$$a_0 = 1.73 \times 10^{-8} \left( \frac{10^5 \text{cm}}{R^2} \right)^2 \frac{10^{-5} \text{eV}}{m} .$$

Using these formulae we rewrite $W$ such that

$$W = 1.2 \times 10^{26} \text{erg} / s \frac{\epsilon_0}{\nu} \frac{c^2}{\nu_0 / \text{cm}^2 s^{-1}} \frac{M}{10^{-9} M_\odot} \frac{B^2}{(1 G)^2} .$$

Here length scales of the media are assumed to be larger than the radius of the axion star $R$, the whole of the star is involved in the media. On the contrary when the scale $L$ of the medium is smaller than $R$, we need to put a volume factor of $(L/R)^3$ on $W$.

We would like to point out that although the electric field $E = c a B / f \pi$ is much small as expected, the energy dissipation $W$ is sufficiently large. The reason is that such a large value of the energy dissipation associated with the axion star is resulted from the large volume of the axion stars; the dissipation arises over the volume.

In order to evaluate the value of $W$, we need to know the mass of the axion star realized in the Universe. According to a creation mechanism of the axion star by Kolb and Tkachev [6–8], some of miniclusters are contracted gravitationally to the axionic boson stars. As the mass of the minicluster has been shown to be typically $10^{-12} M_\odot$, we consider such an amount of the mass $M$ as an order of a scale. Furthermore we need to know how frequently the axion stars collide with stars like the sun and white dwarfs, which are taken as the media as explicit examples. The rate of the encounter is easily obtained by assuming that the halo (its local density $\sim 5 \times 10^{-25}$ g/cm$^3$) in our galaxy is composed of the axion stars whose velocities $\sim 10^{-3}$ are produced gravitationally. For example, the rate of the encounter of an axion star with stars like the sun is approximately once per $10^7 (M/10^{-13} M_\odot)$ years: We note that the cross section of the collision is given by that of the sun because the radius of the sun (\(7 \times 10^{10}\) cm) is much larger than that of the axion star whose relevant mass $M$ is larger than $10^{-15} M_\odot$. The rate is so large that groups researching microlens effects [16] may observe the encounter if the energy dissipation is sufficiently large.

Now let us discuss how large the energy of the axion star is released in a magnetized conducting medium. First we take the sun which is a typical star with a strong magnetic field $10^3 \sim 10^4$ G its con-
vection zone. Assuming the depth of the convection zone being \( \sim 2 \times 10^{10} \) cm, and the magnetic diffusivity \( \nu_\alpha \sim 10^7 \text{cm}^2\text{s}^{-1} \) [17], it follows that

\[
W = 3.2 \times 10^{22} \text{erg} / \left( \frac{B^2}{(5 \times 10^9 \text{G})^2} \right) \frac{M}{10^{-13} \text{M}_\odot},
\]

(6)

where we have set \( c^2 = 1 \). Hence the total energy released from the axion star passing through the sun is given such that

\[
W_i = 4 \times 10^{10} \text{cm} \times W / v = 4.3 \times 10^{25} \text{erg} \left( \frac{B^2}{(5 \times 10^9 \text{G})^2} \right) \frac{M}{10^{-13} \text{M}_\odot},
\]

(7)

where the velocity, \( v \), of the axion star is assumed to be \( 3 \times 10^5 \text{cm/s} \); this is a value obtained by equating a kinetic energy with a gravitational one of the axion star in our galaxy. We have assumed nonexistence of the magnetic field in the radiation zone. Therefore, the effect of the collision of the axion star with a star like the sun is difficult to be observed; remember that the luminosity of the sun is the order of \( 10^{33} \text{erg/s} \). Assuming a larger mass of the axion star leads to larger dissipation of the energy whose effects may be observable, but it leads to lower rate of its collision with a star. Thus as far as we are concerned with such stars with the same physical parameters as those of the sun, it is difficult to detect the effect of such collisions.

In the above discussion we have assumed that the axion star pass through the sun and that it is never trapped to the sun. It is however crucial for the detection whether or not it is trapped to the sun. If the axion star is trapped and looses the whole energy in the sun, the amount of the energy reaches \( \sim 10^{41} \text{erg} \) \((M/10^{-15} \text{M}_\odot)\). Noting that the velocity of the convection is approximately \( 10^5 \) cm/s in the sun, it takes several days or several ten days for the energy released to be transported to the surface of the sun. Thus the resultant luminosity is comparable with, or larger than one of the sun. Such a collision with a star occurs roughly once per \( 10^7 \) year \((M/10^{-15} \text{M}_\odot)\). Then approximately \( 10^5 (M/10^{-15} \text{M}_\odot) \) times collisions per year occur in our galaxy. Therefore, if the axion star is trapped to stars like the sun, the radiation from them may be detectable in a search like MACHO searches. So, it is quite interesting to analyze the dissipation of the kinetic energy of the axion star in the sun and to discuss whether or not it is trapped to the sun.

Next we go on to discuss the case of white dwarfs which may possess strong magnetic field \( \sim 10^6 \text{G} \) with their typical radius \( L \sim 10^8 \) cm. According to recent observations of gravitational microlensing, the population of the white dwarfs has been estimated to be \( 2 \times 10^{11} \text{M}_\odot / 0.5 \text{M}_\odot \sim 4 \times 10^{11} \) within 50 kpc in the halo around our galaxy when their typical mass is 0.5 \( \text{M}_\odot \). Since the number is so large, the collision with the axion stars is expected to occur frequently in our galaxy. Furthermore, as we will show below the axion star deposits the energy on the white dwarf so much that the effect of the dissipation may be observed by a search like MACHO searches. Actually the release of the energy in such a collision is given such that

\[
W = 2.7 \times 10^{22} \text{erg} / \left( \frac{c^2}{\nu_\alpha / \text{cm}^2\text{s}^{-1}} \right) \left( \frac{M^4}{10^{-11} \text{M}_\odot} \right) \left( \frac{B^2}{10^9 \text{G}} \right)^2 \left( \frac{m^6}{10^{-5} \text{eV}} \right),
\]

(8)
where we have taken account of the volume factor, \((L/R)^3 \sim 10^3\) \((M/10^{-14}M_\odot)^3 m_e^6\), because the radius \(R\) of the axion star is larger than the radius \(L\) of the white dwarf. Here we expect that \(\nu_a\) is much smaller than ones of normal metals because the number density of degenerate electrons in the white dwarf is much larger than that of the metals. Actually according to theoretical evaluations [18] we see that \(\nu_a \sim O\left(10^{-4}\right)\) \(\text{cm}^2/\text{s}\) in the case of a crystallized white dwarfs with temperature \(\sim 10^5\)\(\text{K}\) and density \(10^6\) \(\text{g/cm}^3\). Although higher temperature leads to larger \(\nu_a\), it is reasonable to assume that \(\nu_a = 10^{-4}\) \(\text{cm}^2/\text{s}\) since our concern is dark white dwarfs with rather low temperature and large population.

Then \(W\) reaches a value more than \(10^{36}\) \(\text{erg/s}\). Such a large value of \(W \sim 10^{36}\) \(\text{erg/s}\) in eq(8) implies that an actual release of the energy is limited because only a part of the axion star swept by the white dwarf melts and releases its energy. Namely, the fraction, \(I^2\nu/I^0 \sim 10^{-5}(M/10^{-14}M_\odot)^3 m_e^6\) of the mass \(\sim 10^{10}\) \(\text{erg} (M/10^{-14}M_\odot) m_e^6\) is dissipated per unit time; \(10^{35}\) \(\text{erg/s} (M/10^{-14}M_\odot) m_e^6\). This release of the energy, \(W_{\text{rad}} \sim 10^{35}\) \(\text{erg/s} (M/10^{-14}M_\odot) m_e^6\), continues until the white dwarf passes through the axion star. Thus total energy deposited by the axion star is \(2R/\nu \times W_{\text{rad}} \sim 10^{38}\) \(\text{erg/s} (M/10^{-14}M_\odot) m_e^6\). Eventually this energy is transmitted into radiations. If the luminosity is roughly \(L_0\) (although we have not yet analyzed precisely the amount of the luminosity resulted from the energy dissipation, maximal luminosity must be constrained by the rate of the energy dissipation \(W_{\text{rad}}\) of the axion star), the increase of the luminosity of the white dwarf continues about for \(10^5\) sec. In this discussion we simply assume that the part of the axion star swept by the white dwarf melts completely owing to the large value of the dissipation \(W\) in eq(8). But in order to see precisely how the axion star melts, we need to analyze the effect of the dissipation on the axion star itself.

We comment that the value of \(W_{\text{rad}}\) does not depend on the detail of the magnetic field strength of the white dwarf. Even the relatively small magnetic field \(\sim 10^5\) \(G\) leads to the same energy dissipation \(W_{\text{rad}} \sim 10^{35}\) \(\text{erg/s}\) of the axion star.

We may estimate the event rate of the collisions in a solid angle, \(5^\circ \times 5^\circ\), for example. We assume that as indicated by the recent observations of gravitational microlensing, the half of the halo is composed of the white dwarfs with mass \(0.5 \times M_\odot\) and that the other half is composed of the axion stars. Total mass of the halo is supposed to be \(\sim 4 \times 10^{11}M_\odot\). The distribution [19] of the halo is taken such that its density \(\propto (r^2 + 3R_c^2)/(r^2 + R_c^2)^2\) with \(R_c = 4\) kpc where \(r\) denotes a radial coordinate with the origin being the center of the galaxy (the final result does not depend practically on the value of \(R_c\)). Then it is easy to evaluate the event rate of the collisions of axion stars and the white dwarfs,

\[
\frac{4}{\text{year}} \times \left(\frac{0.5 \times 10^{-14}M_\odot}{M^3}\right)^2 \left(\frac{10^{-5} \text{eV}}{m^4}\right)^4 \frac{\Omega}{5^\circ \times 5^\circ},
\]

where \(\Omega\) is a solid angle. We have taken into account the fact that the earth is located at about \(8\) kpc from the center of our galaxy; we have counted the rate of the collisions arising in the region from \(8\) kpc to \(50\) kpc. Therefore, it is possible to observe the phenomena associated with the energy dissipation of the axion stars in the white dwarfs, although the rate depends heavily on the mass of the axion stars.
Similarly as the white dwarf, neutron stars possess strong magnetic fields and high electric conductivities. Thus the energy dissipation of the axion star in this magnetized conducting medium is large. But since the radius of the neutron star is about $\sim 10^6$ cm, the actual energy dissipation in the neutron star is much smaller than that in the white dwarf. The reason is that only a fraction of the mass of the axion star, which is a part swept by the neutron star, is relevant for the energy dissipation. Therefore, the rate of the energy release is approximately $10^{30} (10^9 / 10^9)^3 \text{erg/s} (M/10^{-14} M_\odot)^2 m_a^2 \sim 10^{30} \text{erg/s} (M/10^{-14} M_\odot)^2 m_a^2$. Total energy release is $10^{38} \text{erg} (M/10^{-14} M_\odot)^2 m_a^2$. This is too small for the resultant radiation to be detectable. When the axion star is, however, trapped to the neutron star, the energy released is so larger that the effect of the release may be detectable.

Finally we wish to point out a possible mechanism of producing extremely high energy cosmic rays. It is caused by oscillating electric field of the axion star, $\alpha a E / \pi \sim 10^{-3} B a$, in a tube of a magnetic vortex in a superconducting medium of a neutron star. The superconductor is resulted from the condensation of the proton pairs in a neutron star. Thus the radius of the magnetic flux tube is very small; it is order of 1 GeV $^{-1} \sim 10^{-14}$ cm. Then, we find that the strength of the magnetic field is extremely larger. It is roughly $\pi/e (10^{-14} \text{cm})^2 \sim 10^{20}$ G; $e$ is the charge of the proton. Therefore, an energy acquired by a proton or an electron from the oscillating electric field in a magnetic flux tube is order of $10^{-3} a_s B m^{-1} \sim 10^8 \text{GeV} (M/10^{-10} M_\odot)^2 m_1^2$; $m^{-1}$ is a length which the charged particles run with the light velocity in a cycle $m^{-1}$ of the oscillating electric field. This would be one of mechanisms producing the extremely high energy cosmic rays.

As explained in above examples, the axion stars are possible sources for generating energies in magnetized conducting media. We may apply the idea to systems such as accretion disks with strong magnetic fields around black holes e.t.c.. Probably the existence of the axion will be confirmed indirectly by observing the phenomena associated with the energy release of the axion star.

In summary, we have shown that the coherent axion stars release their energies in the magnetized conducting media such as stars, or white dwarfs. Among them, radiations from the white dwarfs seems to be detectable since their luminosities and rate of the events are sufficiently large. We have also pointed out a possible mechanism for generating extremely high energy cosmic rays.

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