Monopolistic Trading Company and Optimal Tariffs

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1. Introduction

Although general trading companies (sogo-shosha in Japanese) play a crucial role in Japanese foreign trade, they have never been studied in the literature. There are less than ten monopolistic general trading companies which deal with more than 60% of total import value and more than 50% of total export value. Each general trading company set up before the World War II has a lot of subsidiaries (about one hundred) in the world and deals with anything tradable. 1)

If small producers export their products by themselves, export costs might be prohibitively high because of language barrier, transport costs, different commercial practice, complicated exports procedure, costs for market research and so forth, or because of small scale merit of trading service and no accumulation of export technology. If the small producers take advantage of general trading company to export their commodities, they can supply them all over the world. Thus, even if the producers have comparative advantage in their products, they could not export them unless they make use of general trading company.

Existence of monopolistic general trading companies has been ignored in the trade theory because general trading companies has existed only in Japan and Japan had been a relatively small country in the world economy. The major object of the present paper is to develop a systematic approach to deal with a variety of issues deriving from monopolistic trading company.

After introducing the assumptions adopted in the paper in section 2, we deal with a small country model in section 3 and show a free trade equilibrium which resembles the case for tariff revenue maximization as there were no trading company. In the same section we consider existence of tariff and explore the optimal tariff rate which will be shown as a negative value (i.e. subsidy). Section 4 is devoted to a large country model with which we show a free trade equilibrium, consider existence of tariffs, explore the optimal

1) See Patrick and Rosovsky (1976).
subsidy rates, Nash and Stackelberg equilibria, and the first–best solution. Especially we show that there could exist a case where two trading countries adopt subsidy policy at a Nash equilibrium and that the Stackelberg equilibrium, as a home country of the trading company is a follower, is preferable to the Nash equilibrium for the both countries. In section 5 we inspect two trading company model where Nash equilibria under free trade, quantity control and tariff policy are described. In particular we show that the Nash equilibrium under quantity control satisfies the first–best condition.

2. Assumptions

In what follows we assume that
(1) there is a monopolistic trading company which buys exportable commodities from a lot of competitive producers in the home market and sells them in the world market, and at the same time buys importable commodities in the world market and sells them to a lot of competitive consumers in the home market,
(2) unit cost of trading service of the company is so negligible that we can avoid such a kind of problem of transport costs to focus only on monopolistic rents,²)
(3) export costs are prohibitively high if a competitive producer exports its products by itself and import costs are prohibitively high if a consumer imports foreign products by itself,
(4) in a small country model the home country is too small to change the international price but large enough to enjoy a scale merit of trading service with local language and complicated domestic commercial practice which prohibit from approaching the home market by any other foreign trading companies,³)
(5) in a large country model the home country is a large enough to change the foreign price, and
(6) there are two kinds of commodities which are the first (importable) and the second (exportable), and the second commodity is a numéraire.

3. Small Country Model

3.1. Free trade equilibrium

In this model here a country has a local language and special trade practice with which a trading company enjoys monopoly of trading (imports and exports) service market of the country, and which prevent any other foreign trading companies from competing with the company in the market.

Let p and p* be the domestic relative price of the first commodity in the home and the foreign countries respectively. Then, the profit of the company in terms of the second

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²) See Casas (1983) as for the transport cost problem in international trade. In the present paper, in order to focus on the problem of monopolistic trading company, we ignore transport costs.
³) These are sort of natural non-tariff barrier in trading service.
commodity $R$ will be \(^4\)

\[ R = (p - p^*) M. \]

where $M$ denotes import volume of the first commodity. Since the company is a monopolistic supplier of imports and every domestic supplier is a price-taker, the company can control either the domestic price or the import volume. Then, the first-order condition for profit maximization of the company is shown as

\[ dR/dM = p - p^* + M dp/dM = 0 \text{ or } e = 1, \]

where

\[ e = -(dM/dp)p/M = \text{total price elasticity of imports}, \]
\[ r = (p - p^*)/p = \text{profit rate}, \]

and $e > 1$ since $r < 1$. Note that the profit rate is strictly positive as long as $dM/dp < 0$ in \([2]\). Thus, the home price is given as

\[ p = p^* - M dp/dM = p^*/(1 - 1/e) > p^*, \]

and the profit is written as

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4) Let us define the profit of the trading company as follows:

\[ \text{Profit} = (p_1 - p_1^*)(D_1 - S_1) + (p_2^* - p_2)(S_2 - D_2) \]

where

\[ D_i = \text{home demand for } i\text{-th commodity}; i = 1, 2, \]
\[ S_i = \text{home supply of } i\text{-th commodity}; i = 1, 2, \]
\[ p_1 = \text{home domestic price of } i\text{-th commodity}; i = 1, 2, \]
\[ p^*_i = \text{foreign domestic of } i\text{-th commodity}, i = 1, 2. \]

Insert the following condition for trade balance into the definition of the profit above

\[ p_1^*(D_1 - S_1) = p_2^*(S_2 - D_2) \]

to obtain

\[ \text{Profit} = p_1(D_1 - S_1) - p_2(S_2 - D_2). \]

Then, divide the trade balance and the profit by the price of the second commodity to obtain

\[ p^*(D_1 - S_1) = (S_2 - D_2) \]

and

\[ R = p(D_1 - S_1) - (S_2 - D_2) \]

respectively, so that we have \([1]\).
Note that the profit [4] would equal the maximum tariff revenue if there were no trading company as shown in Figure 1 by the hatched area $R$. For simplicity we assume that import function is linear in Figure 1, where the profit maximizing home price is determined at the intersection between the marginal cost ($p^*$) and the marginal revenue ($p + Mdp/dM$) curves.

Figure 2 also shows the free trade equilibrium where the national income $Y$ is defined as

$$ Y = pS_1 + S_3 + R. $$

In Figure 2 the production point and the demand point are shown by $E_s$ and $E_d$ respectively.

### 3.2 Existence of tariff

Let $t$ denote a tariff rate of the country. Then, the profit will be $^5$

$$ R = \frac{p - (1 + t)p^*}{1 - p^*}M. $$

So, the first-order condition for profit maximization will be

$$ \frac{dR}{dM} = p - (1 + t)p^* + Mdp/dM = 0 \text{ or } e^r = 1, $$

where $e < 1$ since $r' < 1$ such that

$$ r' = \frac{p - (1 + t)p^*}{p} = \text{profit rate with tariff}, $$

and the home price will be

$$ p = p^*(1 + t)/(1 - 1/e), $$

and the profit will be

$$ R = (1 + t)p^*M/(e - 1). $$

Thus, if we define $p'$ as unit cost of imports of the company such that

$$ p' = (1 + t)p^*, $$

then the expressions of [6], [7], [8] and [9] are equivalent to those of [1], [2], [3] and [4] respectively.

When the tariff rate changes, the first-order condition for profit maximization [7] also changes as follows;

$$ \frac{dR}{dM} = \frac{d^2R}{dM^2}dM/dt - p^* = 0. $$

Thus, the change in the imports is written as

$$ \frac{dM}{dt} = p^*/(d^2R/dM^2) < 0, $$

where we assume that the second-order condition for profit maximization is satisfied.

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$^5$) Let $t_1$ and $t_2$ denote import and export tariff rate respectively. Then, the profit will be expressed by

$$ \text{Profit} = \left[ p_1 - (1 + t_1)p^*_1 \right] (D_1 - S_1) + \left[ p_2^* - (1 + t_2)p_2 \right] (S_2 - D_2). $$

Plug $p_2 = p_2^* = 1$, $p_1 = p$, $p_1^* = p^*$, and the trade balance,

$$ p^*(D_1 - S_1) = S_2 - D_3, $$

to obtain

$$ R = [p - (1 + t_1 + t_2)p^*] (D_1 - S_1). $$

Thus, the effect of export tariff $t_2$ on the profit of the company is the same as that of import tariff $t_1$. 

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such that \( d^2R/dM^2 < 0 \).

Therefore, an increase in the home tariff rate decreases the import volume to raise the home price and to reduce the profit as follows:

\[
dR/dt = (dR/dM)dM/dt - p^*M = -p^*M < 0.
\]

### 3.3 Optimal tariff rate

Let \( dy \) denote a change in the real income in terms of the marginal utility of the second commodity. Since the national income and the budget constraint are shown as

\[
Y = pS_1 + S_2 + R + tp^*M = pS_2 + S_2 + (p - p^*)M = pD_1 + D_2,
\]

the first-order condition for welfare maximization is given as

\[
dy/dt = (p - p^*)dM/dt = 0 \text{ or } p = p^*,
\]

where \( dy/dt \) denotes the change in the social welfare in terms of marginal social welfare of the second commodity and the production frontier is given by the following implicit function:

\[
F(S_1, S_2) = 0.
\]

From the first-order condition for profit maximization it we have the following optimal tariff rate;

\[
t = -1/e < 0,
\]

i.e., the optimal trade policy is subsidy whose rate equals the inverse of the total elasticity of imports with a negative sign. Suppose in the profit maximizing home price \( p > p^* \) so that \( t > 0 \). In this case, as shown by the first-order condition for welfare maxi-
mization [12], the home country can raise its welfare by reducing the tariff rate since a reduction in the tariff rate increases the import volume as shown by [10] to decrease the domestic price. Even if $t=0$ or if free trade is the case, the wedge between the domestic and the foreign prices still remains as shown is [3]. Under the optimal subsidy rate the profit will be shown as

$$R = -tpM,$$

i. e., the profit equals the total value of subsidy.\(^9\)

**Proposition 1:** The optimum tariff rate is equal to an inverse of price elasticity of imports with a negative sign, i. e., the optimum trade policy is subsidy, and the total subsidy equals the profit of the trading company since the first-order condition for welfare maximization is such that $p=p^*$.  

Figure 1 and 2 also show the equilibrium with subsidy. In Figure 1 we ignore a shift of the import curve as subsidy is provided. In Figure 2, where the optimum production and demand points are denoted by $E'^*$ and $E^*_1$ respectively, the effect of optimum subsidy resembles the effect of tariff removal when we regard the profit of the company as tariff revenue. Thus, the trade with the optimum subsidy corresponds to a free trade without the company whereas the free trade with the company corresponds to a trade with maximum tariff revenue.

4. Large Country Model

Suppose there are two countries in the world, one of which (Japan) has a monopolistic trading company but the other (US) has no monopolistic trading company.

4.1. Free trade equilibrium

Let an asterisk(*) indicate foreign variable. Then, the trade volume balance is shown as

$$M(p) = X^*(p^*),$$

where $X^*$ denotes foreign exports of the first commodity. Now the profit will be shown as

$$R = pM - p^*X^*,$$

and the first-order condition for profit maximization of the company is as follows;

$$dR/dM = (p + M dp/dM) - (p^* + X^* dp^*/dX^*) = 0,$$

where

$$marginal\ revenue = p + M dp/dM,$$

since

$$dM/dp = dD/dp - dS/dp = \partial D/\partial p + (\partial D/\partial Y)dY/dp - dS_Y/dp = D^* - D_\partial D/\partial Y - dS_\partial Y/dp = D^* - S^* - MD^* < 0.$$  

9) In fact, the Ministry of International Trade and Industry (MITI) of Japan has been financing JETRO (Japan External Trade Organization), which is a non-profit public organization and has a lot of subsidiaries in the world, providing free information about trade opportunity and trade policy of foreign countries to Japanese general trading companies. In addition the Japan Export and Import Bank, which is one of public financial organization, has given low interest rates to trade credits. Furthermore, the Ministry of Finance of Japan has been ignoring tax-haven activity of the companies. These might be sort of subsidy.
monopolistic cost = \( p^* + X^* dp^*/dX^* \).

Then, the first-order condition is reduced to

\[ er = 1 + ep^*/e^*p, \]

where

\( e^* = (dX^*/dp^*)p^*/X^* \) = total price elasticity of foreign exports.

In Figure 3 the equilibrium point is indicated as \( E \). Therefore, the home price will be shown as

\[ p = p^*(1 + t^*)/(1 - 1/e), \]

i.e., the wedge between the home and the foreign prices are greater than that in the small country model [3], and the profit will be

\[ R = (1 + e^*)p^*M/(e - 1). \]

In what follows, as shown in Figure 3, we assume \( e^* > 0 \) so that \( e > 1 \) and \( 0 < r < 1 \) in [17].

4.2. Existence of tariffs

Let \( t^* \) denote a foreign export tariff rate. Then, the profit is shown as\(^{10}\)

\[ R = pM - (1 + t^*)(1 + t^*)p^*X^*, \]

and the first-order condition for profit maximization is shown as

\[ \frac{dR}{dM} = p + Mdp/dM - (1 + t^*)(1 + t^*)p^*(X^* dp^*/dX^*) = 0 \]

or

\[ er'' = 1 + (1 + t^*)(1 + t^*)ep^*/e^*p, \]

where

\[ marginal\ revenue = p + Mdp/dM, \]

\[ marginal\ cost = (1 + t)(1 + t^*)p^*(X^* dp^*/dX^*), \]

and the profit rate;

\[ r^* = [p - (1 + t)(1 + t^*)p^*]/p, \]

so that the profit maximizing home price is shown as

\[ p = p^*(1 + t)(1 + t^*)(1 + 1/e)/(1 - 1/e) > 0, \]

and the profit is shown as

\[ \text{Profit} = \frac{p_1 - (1 + t_1)(1 + t^*_1)p_1}{(D_1 - S_1)} + \frac{p_2 - (1 + t_2)(1 + t^*_2)}{(S_2 - D_2)} - \frac{(1 + t_2)p_2}{(D_t - S_1)} \]

Plug \( p_2 = p_2^* \equiv 1, p_1 \equiv p, p^* \equiv p^*^*, \) and the trade balance;

\[ (1 + t^*)p_1^*(D_1 - S_1) - p_2^*(S_2 - D_2)/(1 + t^*_2) \]

to obtain

\[ R = [p - (1 + t^*)(1 + t_1 + (1 + t_2)(1 + t^*_2))/((D_1 - S_1)). \]

Thus, if \( t_2 = t^*_2 = 0, \) then we have [20] or

\[ R = [p - (1 + t_1)(1 + t^*)p^*]/(D_1 - S_1), \]

while if \( t_2 = t^*_2 = 0, \) i.e., if there were no export tariffs, then we have

\[ R = [p - (1 + t_1 + t^*_2)p^*]/(D_1 - S_1). \]
\[ R = (1 + e/e^*) (1 + t) (1 + t^*) p^* M / (e - 1). \]

In Figure 3 the equilibrium point is indicated as \( E_p \).

### 4.3. Home optimum tariff rate

Now, we want to show the optimal tariff rate of the home country under profit maximization of the trading company. First of all, note that the national income is reduced to

\[ Y = p S_1 + S_2 + R + t (1 + t^*) p^* M, \]

so that the definition of the profit \([20]\) gives the following reduced form of the national income;

\[ Y = p S_1 + S_2 + p M - (1 + t^*) p^* X^*. \]

Calculate \(dy/dt\) given \( t^* \) to obtain\(^{11} \)

\[ dy/dt = p (dM/dp) dp/dt - (1 + t^*) (X^* + p^* dX^*/dp^*) dp^*/dt \]

\[ = [p - (1 + t^*) (p^* + X^* dp^*/dX^*)] (dM/dp) dp/dt = 0, \]

where

\[ \text{marginal loss} = -p (dM/dp) dp/dt, \]

\[ \text{marginal benefit} = -(1 + t^*) (X^* + p^* dX^*/dp^*) dp^*/dt, \]

i.e., the marginal loss represents a decrease in the import volume due to an increase in the home price by raising the tariff rate and the marginal benefit comes from a reduction in payment to the foreign country. Thus, the optimal tariff rate must satisfy the following condition;

\[ p = p^* (1 + t^*) (1 + 1/e^*). \]

In Figure 3 the point \( E_p \) satisfies \([26]\). Therefore, from the profit maximizing home price \([22]\) the optimal home tariff rate is given as

\[ t = -1/e, \quad -1 < t < 0. \]

Thus, the expression of the optimal tariff rate does not depend upon country size as recognized by comparing \([27]\) with the rate for a small country \([13]\). Then, the profit under the optimal tariff rate is

\[ R = - (t - 1/e^*) (1 + t^*) p^* M - t (1 + t^*) p^* M = \text{total subsidy}, \]

which is greater than the total subsidy.

**Proposition 2:** In a large country model the expression of the optimum tariff rate is the same as that in a small country model, and the total subsidy is less than the profit of the trading company.

Suppose the government would give the total subsidy as much as the profit of the company such that

\[ R = - t (1 + t^*) p^* M. \]

Then, from the definition of the profit \([20]\) we notice that

\[ p = (1 + t^*) p^*, \]

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11) Differentiating \([15]\) with respect to the tariff rate \( t \), we obtain

\[ dp^*/dt = (dM/dp) (dp/dt)/(dX^*/dp^*). \]

Plug \( dp^*/dt \) into the first equation of \([25]\) to obtain the second.
i.e., under such a situation the domestic price equals the import cost. Thus, from the first-order condition for welfare maximization [25] we obtain
\[ dy^*/dt^* = -(1 + t^*)(p^*/e^*)dM/dt > 0, \]
which implies that the policy is over-subsidy. In this situation the government can raise the welfare by decreasing the subsidy rate to reduce the foreign domestic price. Such a procedure increases the profit of the company and decreases the total subsidy, that is the implication of the second half in Proposition 2.

4.4. Foreign optimum tariff

By the same token, noting that the foreign national income is defined as
\[ Y^* = p^*S^*_1 + S^*_2 + t^*p^*X^*, \]
we obtain the change in foreign real income \( dy^*/dt^* \) with respect to the foreign tariff rate \( t^* \) and the first-order condition for welfare maximization as follows;
\[ [29] \quad dy^*/dt^* = [t^*(X^* + p^*dX^*/dp^*) + X^*]dp^*/dt^* + p^*X^* = 0, \]
where
\[ \textit{marginal loss} = -[t^*(X^* + p^*dX^*/dp^*) + X^*]dp^*/dt^*, \]
\[ \textit{marginal benefit} = p^*X^*, \]
i.e., the marginal loss represents decrease in the export volume and in tariff revenue due to a reduction in the foreign domestic price by raising the tariff rate and the marginal benefit comes from a direct increase in tariff revenue by raising the rate, and from the trade volume balance [25], differentiation with respect to \( t^* \) gives;
\[ [30] \quad dp^*/dt^* = (dM/dp)(dp/dt^*)/(dX^*/dp^*) < 0, \]
where \( dp^*/dt^* < 0 \) since from [21] we have the following change in the first-order condition for profit maximization of the company as the foreign tariff rate changes;
\[ d(dR/dM)/dt^* = (d^2R/dM^2)(dM/dp)dp/dt^* - (1 + t^*)(p^* + X^*dp^*/dX^*) = 0, \]
so that the change in the home price as the foreign tariff rate changes is shown as
\[ [31] \quad dp/dt^* = (1 + t^*)(p^* + X^*dp^*/dX^*)/(d^2R/dM^2)(dM/dp) > 0, \]
where \( dp/dt^* > 0 \) from the second-order condition \( d^2R/dM^2 < 0 \). In the first-order condition for foreign welfare maximization [29] an increase in foreign tariff rate, which raises foreign tariff revenue (marginal benefit), directly reduces the foreign price to decrease the foreign welfare (marginal loss). Thus, the foreign optimal tariff rate is expressed as
\[ [32] \quad t^* = [p/e(dp/dt^*) - 1/e^*]/(1 + 1/e^*) = p/e(1 + 1/e^*)dp/dt^* - 1/(1 + e^*), \]
Since \( dp/dt^* > 0 \) in [32] from [31], the foreign optimal tariff rate could take both signs. If the foreign elasticity of exports \( e^* \) is extremely large relative to the home elasticity of imports \( e \), then the foreign optimal tariff rate might be positive while if the home elasticity of imports \( e \) is extremely large relative to the foreign elasticity of exports \( e^* \), then it might be negative. In fact, if \( t^* = 0 \), we have
\[ dy^*/dt^* = X^*dp^*/dt^* + p^*X^* = [1 - e(dp/dt^*)/pe^*]p^*X^* \]
\[ = [1 + (1 + t^*)(1 + 1/e^*)p^*/Me^*(d^2R/dM^2)]p^*X^*, \]
from [29], [30] and [31], where the second-order condition for profit maximization is calculated as

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\[ \frac{d^3 R}{dM^2} = -2p/eM + Md^2 p/dM^2 < 0. \]

Thus, if the foreign elasticity of exports \( e^* \) is extremely large relative to the home elasticity of imports \( e \), then a reduction in the foreign domestic price by imposing tariff \( dp^*/dt^* \) is fairly small so that \( dy^*/dt^* > 0 \) at \( t^* = 0 \). On the other hand, if the home elasticity of imports is extremely large relative to the foreign elasticity of exports \( e^* \), then a reduction in the foreign domestic price by imposing tariff \( dp^*/dt^* \) is fairly large so that \( dy^*/dt^* < 0 \) at \( t^* = 0 \).

**Proposition 3:** If the foreign elasticity of exports is fairly large relative to the home elasticity of imports, then the foreign optimal tariff rate is positive. Adversely, if the home elasticity is fairly large relative to the foreign elasticity, then the rate is negative.

### 4.5. Nash equilibrium

Suppose each country determines each optimal tariff rate as if other country’s tariff rate is given. Then, from [25] when the foreign tariff rate changes, the first-order condition for home welfare maximization will change as follows:

\[ d(dy/dt)/dt^* = (d^2y/dt^2)(dt/dt^*) - (1 + 1/e^*)p^*(dM/dp)dp/dt = 0 \]

and from [21] the first-order condition for profit maximization of the company changes when the home tariff rate changes as follows:

\[ d(R/dM)/dt = (d^2R/dM^2)(dM/dp)dp/dt - (1 + t^*)(p^* + X^* dp^*/dX^*) = 0. \]

Then, we have the following change in the home price as the home tariff rate changes:

\[ dp/dt = (1 + t^*) (p^* + X^* dp^*/dX^*) / (d^2R/dM^2)(dM/dp) > 0, \]

so that the slope of the home reaction curve is given as

\[ dt/dt^* = (1 + t^*) [(1 + 1/e^*)p^*]^{2/(d^2y/dt^2)}(d^2R/dM^2) > 0, \]

where we assume that the second-order condition for home welfare maximization with respect to the home tariff rate is satisfied such that \( d^2y/dt^2 < 0 \).

Similarly from the first-order condition for foreign welfare maximization [29], we obtain the slope of the foreign reaction curve as follows:

\[ d(dy^*/dt^*)/dt = (d^2y^*/dt^2)(dt^*/dt) + (1 + t^*(1 + e^*))d(dp^*/dt^*)/dt = 0, \]

where from [31] the last derivative term above is reduce to

\[ d(dp^*/dt^*)/dt = d[(dM/dp)(dp/dt^*)/(dX^*/dp^*)] = p^*(1 + 1/e^*)e^*X^*(d^2R/dM^2) < 0. \]

Then, from the foreign optimal tariff rate [32] the slope of the foreign reaction curve is given as

\[ dt^*/dt = - [t^*(1 + 1/e^*) + 1/e^*] (1 + 1/e^*)p^*/(d^2R/dM^2) (d^2y^*/dt^2) \]

\[ = p^* M/(1 + t^*) (d^2y^*/dt^2) < 0. \]

Thus, we have the following benchmark case in Figure 4 where the both reaction curves happen to intersect on the vertical axis of the home optimal tariff rate, then at the Nash equilibrium the foreign optimal tariff rate happens to be zero.\(^{12} \)

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\(^{12} \)In this benchmark case, the home price is

\[ p = p^*(1 + 1/e^*) \]
equilibrium there could be a case where the both countries adopt subsidy policy. From Propositions 3 and 2 we deduce the following:

**Proposition 4**: If the price elasticity of home imports is fairly large relative to that of foreign exports, then there might exist a Nash equilibrium where the both countries adopt subsidy policy.

### 4.6. First-best solution

The first-best solution is characterized by

\[ dy + dy^* = 0. \]

Thus, the first-best condition is given as

\[ dy + dy^* = (p - p^*)dM = 0 \] or \[ p = p^* \]

which is the same as [12]. The problem here is if the condition [37] is realized by appropriate tariff (subsidy) rates in market economies. With profit maximization of the company [22], in order to satisfy the condition above the both countries should set the tariff rates in market economies as follows;

\[ 0 < t(1+t^*)/(1+1/e) < 1, \]

where at least one tariff rate must be negative or at least one country should adopt subsidy policy. With this solution the profit equals the total amount of subsidies such that

\[ R = -(t + t^* + tt^*)pM. \]

For example, a possible combination of subsidies which satisfies [37] is given as

\[ t = -1/e \] and \[ t^* = -1/(1+e^*) \]

since

\[ (1-1/e)/(1+1/e^*) = (1-1/e)^{-1}/(1+1/e^*). \]

In noncooperative situation this combination is, however, unstable since \( dy/dt = 0 \) from the first-order conditions for profit maximization [22] and for welfare maximization [25], but

\[ dy^*/dt^* = p^*X^* > 0 \]

from the first-order condition for foreign welfare maximization [29] and the foreign optimal tariff rate [30]. On the other hand, if the home tariff rate \( t \) is set so as to satisfy

\[ t^* = 0 \] and \( t = -1/e \), so that the profit is such that

\[ R = (1+1/e)p^*M/e. \]

However, in general case, from [22], [27] and [32] the home price is such that

\[ p = p^*[1+p/e(dp/dt^*)] > p^* \]

and the profit is such that

\[ R = (1/e^*)[1+p/e(dp/dt^*)]p^*M/(1+1/e^*). \]

The stability condition for the Nash equilibrium will be such that in absolute value the slope of home reaction curve is flatter than that of foreign reaction curve, as shown in Figure 4.
the foreign optimal tariff rate \([31]\), then \(dy*/dt^*=0\) but \(dy/dt>0\) since \(t^*=-1/(1+e^*)\) and \(-1/e > t\) from the cooperative condition \([38]\). Apparently, at any Nash equilibrium the first-best condition is not satisfied.

**Proposition 5:** In a large country model the first-best condition is such that \(p=p^*\) and at any Nash equilibrium of optimal tariffs the condition is not satisfied.

### 4.7. Indifference curves

In order to make clear the preceding results, we introduce indifference curves as follows: First of all, differentiate the definition of the national income \([24]\) with respect to the tariff rates \(t\) and \(t^*\) to obtain

\[
[41] \quad dy = [p - (1+t^*)p^*]dM - [(1+t^*)dp^* + p^*dt^*]M,
\]

where

\[
\begin{align*}
dM &= (dM/dp)[(dp/dt)dt + (dp/dt^*)dt^*], \\
dp^* &= (dp^*/dt)dt + (dp^*/dt^*)dt^*.
\end{align*}
\]

Set \(dy\) zero to obtain the slope of home indifference curve;

\[
[42] \quad dt/dt^* = p^*/(dy/dt) - (1+t)/(1+t^*),
\]

where from \([31]\) and \([33]\)

\[
(1+t)/(1+t^*) = (dp/dt^*)/(dp/dt).
\]

Then, in Figure 4 on the reaction curve, the slope is infinite since \(dy/dt=0\). When \(t>-1/e\) or the tariff rate takes a value above the reaction curve, the slope is negative from the profit maximizing home price \([22]\) and the first-order condition for welfare maximization \([25]\) at neighborhood of the reaction curve since \(p>(1+t^*)(1+1/e^*)p^*\) so that \(dy/dt<0\). Adversely, when \(t<-1/e\), the slope is positive at neighborhood of the curve. Thus, we have an indifference curve \(U_t\) shown in Figure 4. Intuitively, the foreign subsidy is favorable to the home welfare. Similarly, the slope of foreign indifference curve is given as

\[
[43] \quad dt/dt^* = 1/[p^*X^*/(dy*/dt^*) - 1].
\]

Then, on the reaction curve, the slope is zero since \(dy*/dt^*=0\). When the foreign tariff rate \(t^*\) takes a value just above the reaction curve, the slope is negative from the first-order condition of welfare maximization \([29]\) while when \(t^*\) takes a value just below the curve, the slope is positive. Thus, we have an indifference curve \(U_f\) shown in Figure 4. Intuitively, the home subsidy is favorable to the foreign welfare. Therefore, if the foreign country is a leader, the Stackelberg equilibrium could be given by such a point \(S_f\) shown in Figure 4, where the slope of home reaction curve \([34]\) equals that of foreign indifference curve \([43]\). At the equilibrium the foreign country, as well as the home country, is better off than at the Nash equilibrium \(N\). However, if the home country is a leader the home country’s welfare is improved while the foreign country’s welfare is deteriorated at such a point \(S_h\).

**Proposition 6:** If the foreign country is a leader, the Stackelberg equilibrium is preferable to the Nash equilibrium for the both countries, while if the home country is a leader, the Stackelberg equilibrium is preferable to the Nash equilibrium only for the home
country.

The cooperative solution with profit maximization of the trading company could be found at such a point $F$ in the hatched area of Figure 4, where from the optimal tariff rates [27] and [32], and the cooperative tariff rates [38], we have

$$t < -1/e; \frac{p}{e}(dp/dt^*)(1+1/e^*) - 1/(1+e^*) > t^* > -1/(1+e^*).$$

**Proposition 7:** The cooperative solution could be realized on the southwest of the Nash equilibrium in market economies with higher subsidy rates than the noncooperative rates.

5. Two Trading Company Model

Recently the Korean government has fostered general trading companies in order to compete with Japanese trading companies to extract their monopolistic rents in trading service market. In this section we assume that there are two trading companies, one of which belongs to the home country (Japan) and the other belongs to the foreign country (Korea).

5.1. Free trade equilibrium

Now, the profit of the home company is shown as\(^{13}\)

$$R = (p - p^*)M',$$

where $M' \equiv (M - M^*)$ represents home imports dealt by the home company, $M$ total home imports and $M^*$ home imports dealt by the foreign company. In this model we consider a case where the home (foreign) company can control only its import volume given the home imports dealt by the foreign (home) company. Then, given $M^*$, the first—order condition for profit maximization of the home company is given as

$$dR/dM' = p - p^* + M'(dp/dM' - dp^*/dM') = 0.$$  

By the same token, we obtain the first—order condition for profit maximization of the foreign company. First of all, the profit of the foreign company is similarly

$$R^* = (p - p^*)M^*.$$  

Then, given $M'$, the first—order condition for profit maximization of the foreign company is

$$dR^*/dM^* = p - p^* + M^*(dp/dM^* - dp^*/dM^*) = 0.$$  

Figure 5 shows the home company’s equilibrium at $E$, given $M^*$.

5.2. Nash equilibrium under free trade

Now, from the two first—order conditions for profit maximization, [46] and [48], we

---

\(^{13}\) Let $X_2$ and $X_1^*$ be home exports dealt by the home and the foreign companies respectively so that $X_2 = (X_2^* + X_2)$ is the total home exports: Then,

home profit $= (p_1 - p_1^*)M^* + (p_2 - p_2^*)X_2$,

home trade balance; $p_2^*M^* = p_2X_2$ or $pM^* = X_2$.

Thus, we have [45], $R = (p - p^*)M'$. Similarly,

foreign profit $= (p_1 - p_1^*)M^* + (p_2 - p_2^*)X_1^*$,

foreign trade balance; $p_1M^* = p_2X_2$ or $pM^* = X_2$,  

then, we have [47], $R^* = (p - p^*)M^*$ below.
obtain the following two reaction curves:

\[ M' = (p - p^*)/(dp^*/dM' - dp/dM') \]
\[ M^* = (p - p^*)/(dp^*/dM^* - dp/dM^*) \]

If the two reaction curves are symmetric, we necessarily have \( M' = M^* \) as a Nash equilibrium. The reaction curves [49] and [50] are, however, asymmetric in general. So, we want to show \( M' > M^* \) if \( e^* > 0 \) assumed in subsection 4.1, or if the compensated elasticity of foreign exports \( e^*_\alpha \) is greater than the marginal propensity to consume the first commodity in the foreign country \( m^* \).

First of all, let us show the value of the denominators of [49] and [50]: Given \( M^* \), the total derivative of imports with respect to the home price will be

\[ dM'/dp = dM/dp = aM/\partial p + (aD_1/\partial Y)dY/dp, \]

where

\[ aM/\partial p = aD_1/\partial p - aS_1/\partial p = -e^* M/p - mD_1/p, \]
\[ e^*_\alpha = \text{compensated elasticity of home imports}, \]
\[ m = p_\alpha D_1/\partial Y = \text{marginal propensity to consume the first commodity}, \]
\[ dY/dp = S_1 \text{ since } dR/dp = 0. \]

Then, inverse of the second term of the denominator in [49] will be

\[ dM'/dp = -eM/p < 0, \]

where \( e = e^*_\alpha + m \). On the other hand, the total derivative of foreign exports with respect to the foreign price will be

\[ dM'/dp^* = dX^*/dp^* = aX^*/\partial p^* - (aD_1^*/\partial Y^*)dY^*/dp^*, \]

where

\[ aX^*/\partial p^* = aS_1^*/\partial p^* - aD_1^*/\partial p^* = e^*_\alpha X^*/p^* + m^* D_1^*/p^*, \]
\[ e^*_\alpha = \text{compensated elasticity of foreign exports}, \]
\[ m^* = p^* aD_1^*/\partial Y^* = \text{marginal propensity to consume the first commodity}, \]
\[ dY^*/dp^* = S_1^* + dR^*/dp^*, \]

where since \( M^* \) is fixed

\[ dR^*/dp^* = (dp/\partial p^* - 1)MF < 0, \]

and from the trade volume balance [15],

\[ dp/\partial p^* = (dX^*/\partial p^*)/(dM/\partial p) = (dM'/\partial p^*)/(dM'/\partial p) < 0. \]

Then, inverse of the first term of the denominator in [49] will be

\[ dM'/dp^* = Me(e^* + s^* m^*)/(p^* e - ps^* m^*), \]

where \( e^* = e^*_\alpha - m^* \), and \( s \) and \( s^* \) denote the share of home imports dealt by the home and the foreign company respectively, i.e., \( s = M'/M^* \) and \( s^* = M^*/M^* \).

Similarly, noting that \( dR/\partial p = (1 - dp/\partial p)M' > 0 \) given \( M' \), we obtain
\[ \frac{dM^*}{dp^*} = \frac{e^*M}{p^*} > 0, \]
\[ \frac{dM^*}{dp} = -Me^*(e-sm)/(pe^*+p^*sm) < 0. \]

Then, if \( e^* > 0 \), we have
\[ \frac{M^*}{M} = \frac{e^*(e+sm^*)}{e^*(e-sm)} > 1. \]

The difference between the denominators of [49] and [50] rises from difference in income effects through changes in the profit of the companies. In [49] \( M^* \) is given whereas in [50] \( M' \) is given. Hence, subtraction the first term of the denominator in [50] from that in [49] makes
\[ \frac{dp^*/dM^* - dp/dM^*}{dM^*} = -s^*m^*(e^* + e^*p)/Me^*(e^* + s^*m^*) < 0, \]
from [52] and [53], and similarly from [51] and [54],
\[ \frac{dp/dM^* - dp/dM}{dM^*} = -sm(e^* + e^*p)/Me^*(e^* - sm) < 0. \]

In [55] an increase in \( M^* \) (given \( M' \)) raises \( p^* \) if \( e^* > 0 \) and gives rise to no income effect through a change in \( R^* \) (since \( dR^*/dp^* = 0 \)) while an increase in \( M' \) (given \( M^* \)) also might raise \( p^* \) but gives rise to a negative income effect through a decrease in \( R^* \) (since \( dR^*/dp^* < 0 \)) which decreases \( p^* \), so that \( dp^*/dM' < dp^*/dM^* \). On the other hand, in [56] an increase in \( M' \) (given \( M^* \)) reduces \( p \) and gives rise to no income effect through a change in \( R \) (since \( dR/dp = 0 \)) while an increase in \( M^* \) (given \( M' \)) also reduces \( p \) if \( e^* > 0 \) but gives rise to a negative income effect through a decrease in \( R \) (since \( dR/dp > 0 \)) which decreases \( p \), so that \( dp/dM^* < dp/dM' \). Hence, if \( e^* > 0 \) (we have already assumed in subsection 4.1.), then both of [55] and [56] are negative, so that the denominator of [49] is smaller than that of [50] or \( M'^* > M^* \) at the Nash equilibrium \( N_e \), shown in Figure 6. The positive total elasticity of foreign exports (excluding income effect from a change in the profit of the foreign company) \( e^* > 0 \) also implies the inequality \( e^* > m^* \) since \( e^* = e^* - m^* \).

**Proposition 8:** If the compensated elasticity of foreign exports is greater than the marginal propensity to consume the first goods, then the import volume dealt by the home company is greater than that by the foreign company.

Since \( M'^* = 0 \) when \( M^*(M) \) is so large that \( p = p^* \), and since \( M'(M^*) \) takes a volume as a international monopoly when \( M^*(M') = 0 \) as discussed in the preceding section and shown in Figure 3, dominant signs of the reaction curves [49] and [50] are negative. So, in Figure 6 we assume that the both reaction curves slope forward. Suppose \( M'^* = 0 \). Then, the home imports dealt by the home company equals the volume which satisfies the first-order condition for home company’s profit maximization [16]. Given the volume, the foreign company will
deal with positive amount since from the first-order condition for profit maximization of the foreign company [48], noting that \( M^* = 0 \), we have

\[
dR^*/dM^* = p - p^* > 0.
\]

Thus, if the home import is such a volume that \( p \) equals \( p^* \), then \( M^* = 0 \) as a result of profit maximization. Hence, as shown in Figure 6, the foreign reaction curve is drawn above the home reaction curve near around the \( M' \) axis.

5.3. Nash equilibrium under quantity control

Suppose the home government directly controls the import volume dealt by the home company to maximize the home welfare, given \( M^* \). If \( dR/dM' > 0 \) at the controlled volume, the policy implies the import quota. Adversely, if \( dR/dM' < 0 \) at the volume, the policy implies the compulsory quota and if \( R > 0 \) at the volume, subsidy should be given to the company by means of lump-sum tax. Then, since

\[
Y = pS_1 + S_2 + (p - p^*)M' = pD_1 + D_2,
\]

the first-order condition for welfare maximization will be

\[
[57] \quad dy/dM' = dR/dM' - Mdp/dM' = p - M^*dp/dM' - (p^* + M'dp^*/dM') = 0,
\]

where

- marginal benefit = \( p + M'dp/dM' - Mdp/dM' = p - M^*dp/dM' \),
- marginal loss = \( p^* + M'dp^*/dM' \),

i.e., the marginal benefit represents an increase in net revenue and the marginal loss an increase in net payment. As shown in Figure 5, the optimal quantity will be found at \( E_* \) where the both curves of the marginal benefit and marginal loss (cost) intersect each other. Thus, in general, the optimal quantity of imports for welfare maximization is more than that for profit maximization, given \( M^* \). If the equilibrium \( E_* \) happens to correspond to the import volume where \( p = p^* \), then it holds that

\[
M'dp^*/dM' + M^*dp/dM' = 0,
\]

as shown in Figure 5, where the both lines of exports and imports are drawn straight for convenience' sake. Since \( dR/dM' = Mdp/dM' < 0 \) from [51] and [57], the home policy reaction curve is always above the home company’s reaction curve [46], as shown in Figure 6. On the \( M' \) axis, if \( p = p^* \), then from [52] and [57] we have

\[
dy/dM' = -M'dp^*/dM' = -p^*/e^* < 0
\]

so that the reaction curve [57] is below the foreign company’s reaction curve [50] near around the \( M' \) axis as shown in Figure 6.

By the same token, since

\[
Y^* = p^*S^*_1 + S^*_2 + (p - p^*)M^* = p^*D^*_1 + D^*_2,
\]

as for the foreign policy reaction curve we have

\[
[58] \quad dy^*/dM^* = dR^*/dM^* + Mdp^*/dM^* = p + M^*dp/dM^* - (p^* - M'dp^*/dM^*) = 0,
\]

where

- marginal benefit = \( p + M^*dp/dM^* \),
- marginal loss = \( p^* + M^*dp^*/dM^* - Mdp^*/dM^* = p^* - M'dp^*/dM^* \),

i.e., the marginal benefit represents an increase in net revenue and the marginal loss
an increase in net payment. Since $dR^*/dM^* = -Mdp*/dM^* < 0$ from [53] and [58], the foreign policy reaction curve is always above the foreign company’s reaction curve [48]. On the $M^*$ axis, if $p = p^*$, then from [54] and [58] we have $dy*/dM^* = M^*dp/dM^* < 0$ so that the reaction curve [58] is below the home company’s reaction curve [51] near around the $M^*$ axis. Therefore, the both policy reaction curves must have intersection somewhere.

Now, inspect the home policy reaction curve. From [57], the curve is given by:

$$p - p^* = M' dp^*/dM' + M^* dp/dM',$$

where

$$dM'/dp = -e^*_M/p \text{ since } dR/dp = M \text{ from } dR'/dM' = Mdp/dM' \text{ in } [57],$$

$$dM^*/dp^* = e^*_M(e^* + s^* m^*)/(p^*e^*_s - ps^* m^*).$$

Then, we have the following wedge between the both domestic prices on the home policy reaction curve:

$$[59] \quad p - p^* = (s^* pe^*_s - s^* pe^*_s)/e^*_s(e^* + s^* m^*).$$

Similarly, from [58], the foreign policy reaction curve is given by

$$p - p^* = -M^* dp/dM^* - M' dp^*/dM^*,$$

where

$$dM^*/dp = e^*_M/p^* \text{ since } dR^*/dM^* = -M^* dp/dM^* \text{ in } [58],$$

$$dM^*/dp^* = -e^*_M(e - sm)(pe^*_s + p^* sm).$$

Then, we have the following wedge between the both domestic prices on the foreign policy reaction curve:

$$[60] \quad p - p^* = (s^* pe^*_s - s^* pe^*_s)/e^*_s(e - sm).$$

Noting that the numerators in [59] and [60] on the right-hand-side terms are the same in absolute value, we notice that at the Nash equilibrium

$$[61] \quad p = p^* \text{ and } se^*_s = s^* e^*_s,$$

since the denominators in [59] and [60] on the right-hand-side terms are positive such that

$$e^*_s(e^* + s^* m^*) > 0, \quad e^*_s(e - sm) > 0.$$

Since the first-best condition has been given by [37] as

$$dy + dy^* = (p - p^*)(dM' + dM^*) = 0,$$

at the Nash equilibrium the first-best condition is satisfied as shown by [61]. In Figure 6, the contract curve is shown as a negative forty-five degree line since $M' + M^*$ is constant and uniquely determined when $p = p^*$ so that there are no profits in the trading companies. As shown in Figure 6, each policy reaction curve necessarily intersects the contract curve and each other at the same point $N_s$. In fact, at the Nash equilibrium $N$, the both welfares of the countries are maximized since

$$dy/dM^* = (dy/dM) dM/dM^* - M^*dp/dM^* - M'dp/^dM^* = 0,$$

$$dy^*/dM^* = (dy^*/dM^*) dM^*/dM' + M^*dp/dM' + M'dp/^dM' = 0.$$

**Proposition 9:** In a two trading company model, when each government adopts the quantity control policy, the first-best condition is satisfied at the Nash equilibrium.
The import volume of each country is given by the second condition of [61] such that

\[ M'/M^* = e^*_v / e_v. \]

Thus, the relative volume of imports depends upon the compensated elasticities of home imports and foreign exports so that it is ambiguous if \( M' > M^* \) (though \( M' > M^* \) in Figure 6).

5.4. Nash equilibrium under tariff policy

Finally, we would like to show that under tariff (or subsidy) policy the Nash equilibrium does not satisfy the first-best condition. We consider here a two stage Nash equilibrium where the governments seek the optimal tariff rates in a set of Nash equilibria of two trading companies. Now, respective profits of trading companies are shown as:

\[ R = \left[ p - (1 + t)(1 + t^*)p^* \right] M', \]

\[ R^* = \left[ p - (1 + t)(1 + t^*)p^* \right] M^*. \]

Each company maximizes each profit, given the other imports and tariff rates such that

\[ dR/dM' = \left[ p - (1 + t)(1 + t^*)p^* \right] M' + \left[ dp/dM' - (1 + t)(1 + t^*)dp^*/dM' \right] = 0, \]

\[ dR^*/dM^* = \left[ p - (1 + t)(1 + t^*)p^* \right] M^* + \left[ dp/dM^* - (1 + t)(1 + t^*)dp^*/dM^* \right] = 0. \]

The first-order conditions for profit maximization [65] and [66] give one of Nash equilibria with various tariff rates. Each company’s optimal import volume changes according to tariff rates such that

\[ d(dR/dM')/dt (d^2R/dM'^2) dM'/dt - (1 + t^*)(p^* + M^*dp^*/dM^*) = 0, \]

\[ d(dR^*/dM^*)/dt (d^2R^*/dM'^2) dM^*/dt - (1 + t^*)(p^* + M^*dp^*/dM^*) = 0, \]

from [65] and [66], and similarly

\[ d(dR/dM')/dt = (d^2R/dM'^2) dM'/dt - (1 + t)(p^* + M^*dp^*/dM^*) = 0, \]

\[ d(dR^*/dM^*)/dt = (d^2R^*/dM'^2) dM^*/dt - (1 + t)(p^* + M^*dp^*/dM^*) = 0. \]

Each government chooses the optimal tariff (or subsidy) rate so as to maximize its welfare among Nash equilibria of two companies, given the other tariff rate as follows:

Since

\[ Y = pS_1 + S_2 + R + t(I + t^*)p^* M, \]

\[ dy/dt = (1 + t^*)p^* M^* + (1 + t^*)p^* \left[ Mdp^*/dt + tdM/dt \right] - Mdp/dt \]

\[ = (1 + t^*)p^* M^* + [(1 + t^*)(1 + 1/e^*)tp^* + p/e] dM/dt = 0, \]

where

\[ dR/dt = -(1 + t^*)p^* M' \] from [65],

\[ dp^*/dt = (dp^*/dM)dM/dt = (p^*/e^* M)dM/dt, \]

\[ dp^*/dt = (dp^*/dM)dM/dt = -(p/eM)dM/dt, \]

and from [67] and [68]

\[ dM/dt = dM'/dt + dM^*/dt = (1 + t^*) A, \]

where

\[ A = (p^* + M^*dp^*/dM^*)/(d^2R^*/dM'^2) + (p^* + M^*dp^*/dM^*)/(d^2R^*/dM'^2) < 0. \]

Similarly, since

\[ Y^* = p^* S^*_1 + S^*_2 + R^* + t^*p^* M, \]

\[ dy^*/dt^* = p^*(M' - tM^*) + (1 + t^*)Mdp^*/dt^* + t^*p^* dM/dt^* \]

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where
\[ \frac{dR^*}{dt^*} = -(1+t)p^*M^* \text{ from [66]}, \]
\[ \frac{dp^*}{dt^*} = (dp^*/dM)dM/dt^* = (p^*/e^*M)dM/dt^*, \]
and from [69] and [70]
\[ \frac{dM}{dt^*} = M^*/dt^* + M^*/dt^* = (1+t)A. \]

Now, we can show that the two stage Nash equilibrium is not first-best. First of all, calculate \( \frac{dy}{dt^*} \) and \( \frac{dy^*}{dt} \) at the two stage Nash equilibrium as follows:
\[ \frac{dy}{dt^*} = -Mdp/dt^* - (1+t)p^*M^* + t[p^*M^* + (1+t^*)(p^*dM/dt^* + Mdp^*/dt^*)] = 0. \]

Plug [71] into the above to obtain
\[ \frac{dy}{dt^*} = -p^*M, \]
and similarly
\[ \frac{dy^*}{dt} = -p^*M(1+t^*)/(1+t). \]

Thus, we obtain the final proposition in this paper.

**Proposition 10:** In a two trading company model, when each government adopts the tariff (or subsidy) policy, the first-best condition is not satisfied at the two stage Nash equilibrium.

**References**
